A Generalized Model for Coupled Lines and its Application to Planar Balun Synthesis

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Abstract — A network model for unsymmetrical and inhomogeneous coupled lines has been derived based on normal mode parameters. This model is useful for synthesis of single and multi-layer coupled line circuits. As an example, the model is used for synthesis of a planar balun configuration. The synthesis procedure is verified by comparison with the analysis results.

I. Introduction

Coupled lines are widely used in filters, power coupling and impedance matching networks at microwave frequencies. A network model of coupled lines in homogeneous medium has been derived by Malherbe using capacitance matrices and its applications for network synthesis have been reported [1]. However, this model is not valid for coupled lines in inhomogeneous medium where phase velocities of the two modes are not equal. In inhomogeneous medium, not only capacitance matrix but also inductance matrix of coupled lines is needed in the derivation of network models. Another model has been proposed by Chang and Lee for inhomogeneous lines in the cases where the congruence condition is satisfied [2]. But, this condition is not generally true for multi-layer circuits. Therefore, for synthesis of general coupled line circuits, a new network model is needed which does not make assumptions of congruence or homogeneity.

This paper presents the derivation of such a generalized network model for coupled lines. The derivation is based on the four-port impedance or admittance matrix which could be found by using normal mode parameters. Also, in this paper, the new four-port model is used for synthesis of a planar balun circuit [3]. By using this model for planar balun, a good synthesis procedure has been developed. An example is presented and the results are compared with the direct analysis of the circuit.

The final four-port network representation can be simplified for two-port coupled lines by applying different termination conditions on each port. These two-port equivalent networks are useful in synthesis of two-port coupled line circuits (like couplers, filters, etc.) and some of them will be presented at the Symposium.

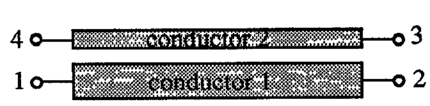


Figure 1. Schematic of a section of uniform coupled lines.

II. Generalized Coupled Line Models

By solving the general coupled telegrapher's equations, two coupled lines (*Figure 1*) could be characterized by two modes (c mode and π mode) using normal mode parameters. These parameters are the different phase velocities ($\mathcal{V}_C$ and $\mathcal{V}_\pi$), the ratios of voltages on the two conductors (R_C and R_π) and the line impedances (Z_{C1}, Z_{C2}, $Z_{\pi 1}$ and $Z_{\pi 2}$) [4]. They can be obtained directly from capacitance and inductance matrices. The Z-matrix of a section of two coupled lines in terms of these normal parameters is shown in Table 1, where θ_C and θ_π are electrical lengths for the two modes of the coupled lines.

Every element in the Z-matrix of *Table 1* is a sum of two terms. One depends on the electrical length for c mode only and the other depends on the electrical length for π mode only. If the Z-matrix is separated into a sum of two matrices, the elements in each one of them depend only on either θ_C or θ_π, then each of these two matrices could be modeled by using one transmission line section and two transformers as shown in *Figure 2*.

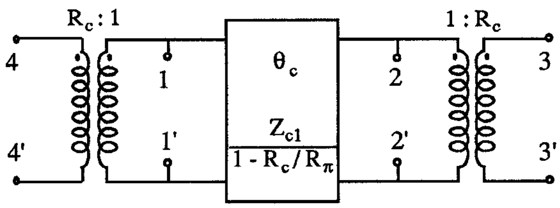


Figure 2. Network model for two matrices whose sum yields Z-matrix of Table 1.

In this network (*Figure 2*), the transmission line section has the electrical length of one mode and its line impedance depends on the characteristic impedance of that mode. The turns ratio of the two transformers is the ratio of voltages on the two conductors of the coupled lines for that mode.

The final four port network model for a coupled line section could be found by connection of these two networks in series. The result is shown in *Figure 3*. If the Y-matrix is used instead of Z-matrix in the derivation of network models, another model could be found by using the same method. These models are exact models (without any approximation) of general coupled lines and they are equivalent to each other.

Also, by application of the different port conditions to the models, the equivalent circuits of a large variety of two-port

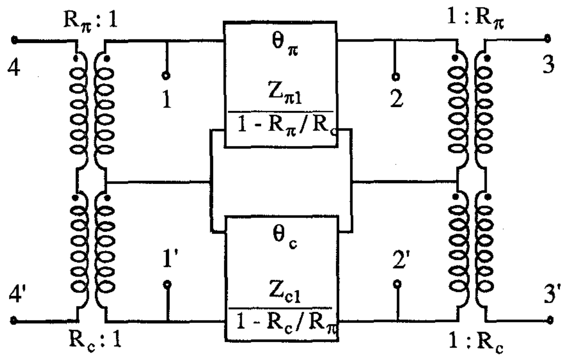


Figure 3. Network model of general coupled lines, based on the Z-matrix.

structures based on coupled lines can be obtained. These two-port models are of the same form as those obtained by Tripathi [5] or Zysman and Johnson [6] using other methods.

III. Synthesis of Planar Baluns

The new four-port network model of coupled lines has been used for designing a two-layer monolithic balun configuration (*Figure 4*) described in [3]. This kind of balun structure is similar but different from Marchand balun [7] in that the transmission line sections in different layers are not isolated from each other. The circuit could be viewed as the connection of two pairs of coupled lines. By using the new network model derived in section II, the result of network representation of this planar balun is shown in *Figure 5*. This network could be reconfigurated by using network theory and expressed as shown in *Figure 6*. This result should be compared with the equivalent circuit of the original Marchand balun [7]. In this equivalent circuit of the planar balun, the transmission line sections have different electrical lengths, which is due to the inhomogeneity of the configuration, whereas in the equivalent circuit of Marchand balun they have the same electrical length. Another difference is that, in the planar balun, the transformers in the equivalent circuit create coupling between transmission line sections.

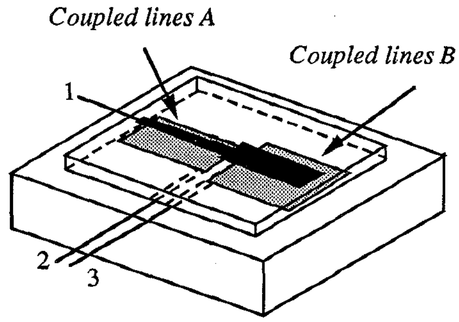
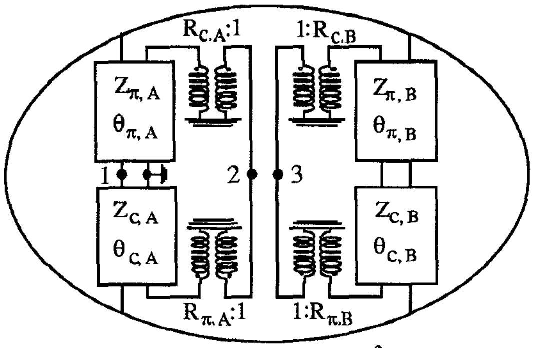


Figure 4. Planar balun; port 1 is the unbalanced port and terminals 2 & 3 constitute the balanced port.



$$Z_c = Z_{c2}(1 - R_\pi / R_c)\,,\; Z_\pi = Z_{\pi 1} R_c^2 (1 - R_\pi / R_c)$$

Figure 5. Network model of planar baluns.

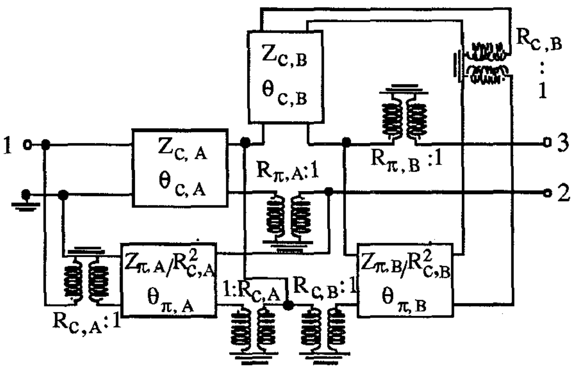


Figure 6. Another equivalent circuit of planar baluns.

$$\begin{bmatrix}
\frac{Z_{c1}\coth\theta_c}{1-R_c/R_\pi}+\frac{Z_{\pi 1}\coth\theta_\pi}{1-R_\pi/R_c} & \frac{Z_{c1}\operatorname{csch}\theta_c}{1-R_c/R_\pi}+\frac{Z_{\pi 1}\operatorname{csch}\theta_\pi}{1-R_\pi/R_c} & \frac{R_c Z_{c1}\operatorname{csch}\theta_c}{1-R_c/R_\pi}+\frac{R_\pi Z_{\pi 1}\operatorname{csch}\theta_\pi}{1-R_\pi/R_c} & \frac{R_c Z_{c1}\coth\theta_c}{1-R_c/R_\pi}+\frac{R_\pi Z_{\pi 1}\coth\theta_\pi}{1-R_\pi/R_c} \\
\frac{Z_{c1}\operatorname{csch}\theta_c}{1-R_c/R_\pi}+\frac{Z_{\pi 1}\operatorname{csch}\theta_\pi}{1-R_\pi/R_c} & \frac{Z_{c1}\coth\theta_c}{1-R_c/R_\pi}+\frac{Z_{\pi 1}\coth\theta_\pi}{1-R_\pi/R_c} & \frac{R_c Z_{c1}\coth\theta_c}{1-R_c/R_\pi}+\frac{R_\pi Z_{\pi 1}\coth\theta_\pi}{1-R_\pi/R_c} & \frac{R_c Z_{c1}\operatorname{csch}\theta_c}{1-R_c/R_\pi}+\frac{R_\pi Z_{\pi 1}\operatorname{csch}\theta_\pi}{1-R_\pi/R_c} \\
\frac{R_c Z_{c1}\operatorname{csch}\theta_c}{1-R_c/R_\pi}+\frac{R_\pi Z_{\pi 1}\operatorname{csch}\theta_\pi}{1-R_\pi/R_c} & \frac{R_c Z_{c1}\coth\theta_c}{1-R_c/R_\pi}+\frac{R_\pi Z_{\pi 1}\coth\theta_\pi}{1-R_\pi/R_c} & \frac{R_c^2 Z_{c1}\coth\theta_c}{1-R_c/R_\pi}+\frac{R_\pi^2 Z_{\pi 1}\coth\theta_\pi}{1-R_\pi/R_c} & \frac{R_c^2 Z_{c1}\operatorname{csch}\theta_c}{1-R_c/R_\pi}+\frac{R_\pi^2 Z_{\pi 1}\operatorname{csch}\theta_\pi}{1-R_\pi/R_c} \\
\frac{R_c Z_{c1}\coth\theta_c}{1-R_c/R_\pi}+\frac{R_\pi Z_{\pi 1}\coth\theta_\pi}{1-R_\pi/R_c} & \frac{R_c Z_{c1}\operatorname{csch}\theta_c}{1-R_c/R_\pi}+\frac{R_\pi Z_{\pi 1}\operatorname{csch}\theta_\pi}{1-R_\pi/R_c} & \frac{R_c^2 Z_{c1}\operatorname{csch}\theta_c}{1-R_c/R_\pi}+\frac{R_\pi^2 Z_{\pi 1}\operatorname{csch}\theta_\pi}{1-R_\pi/R_c} & \frac{R_c^2 Z_{c1}\coth\theta_c}{1-R_c/R_\pi}+\frac{R_\pi^2 Z_{\pi 1}\coth\theta_\pi}{1-R_\pi/R_c}
\end{bmatrix}$$

Table 1. Z-matrix of a section of coupled lines.

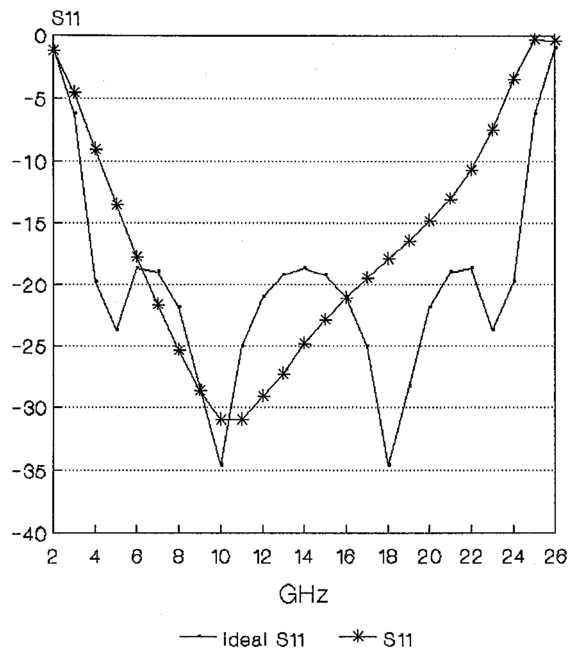


Figure 8. Return loss at the input (unbalanced) port for the synthesized and ideal baluns.

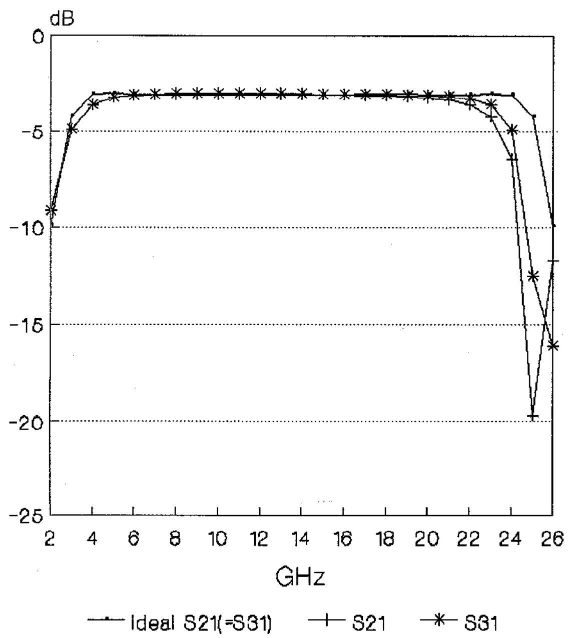


Figure 9. Insertion loss for the synthesized and ideal baluns.

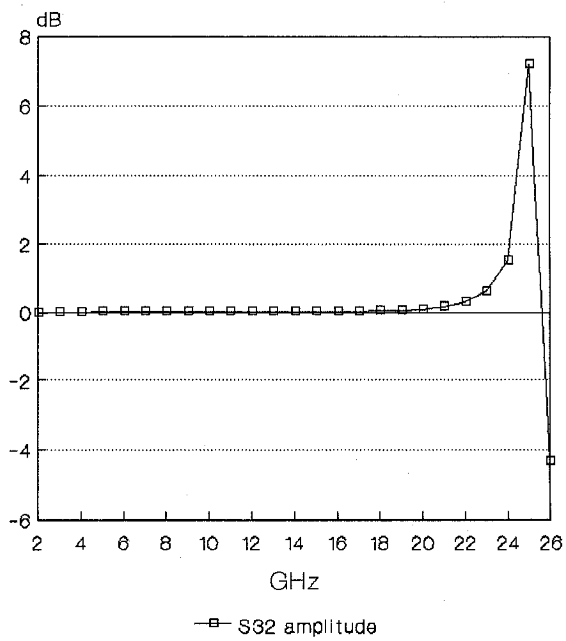


Figure 10. Amplitude unbalance at the output (balanced) port for the synthesized balun.

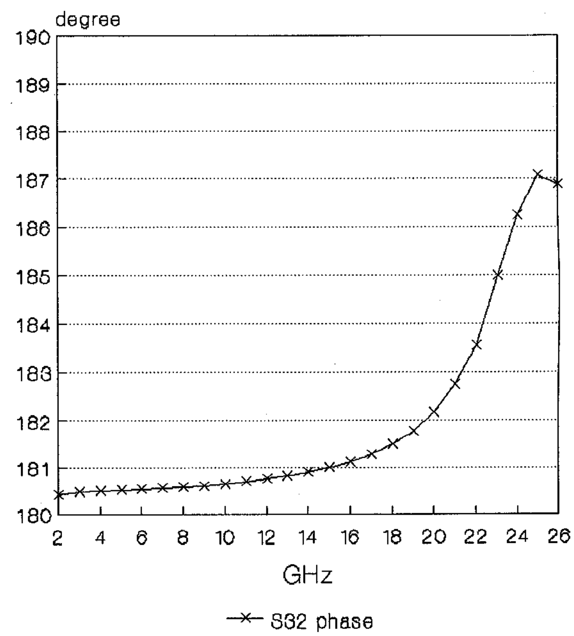


Figure 11. Phase unbalance at the output (balanced) port for the synthesized balun.

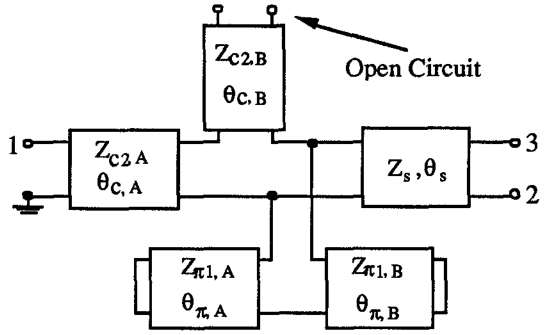


Figure 7. Simplified model of planar baluns.

In *Figure 6*, if $R_\pi \approx 1$ and R_C is large, the transformers in the network could be ignored and this results in a network like the equivalent circuit of the Marchand balun (*Figure 7*), which is more suitable for implementation of the synthesis procedures. Configuration in this figure includes another transmission line section (Z_S,θ_S) corresponding to the planar line at the balanced output port. The synthesis procedures for such a network configuration is available in [8] for the case when the electrical lengths of the transmission line sections are equal.

As an example, a planar balun with bandwidth of 4 to 24 GHz, unbalanced input impedance of 25 ohm and balanced output impedance of 50 ohm, has been designed using 6 mil GaAs substrate with a second dielectric layer of 1.8 μm thick silicon nitride. Using the network model in *Figure 7*, the synthesis procedure in [8] suggests:

$Z_{C2,A}$ = 31 ohm
$Z_{C2,B}$ = 15 ohm
$Z_{\pi1,A} + Z_{\pi1,B}$ = 80 ohm
Z_S = 39 ohm

These parameters are used to synthesize the coupled line sections and the results are:

Coupled Lines A	Coupled Lines B
W_1 = 160 μm	W_1 = 160 μm
W_2 = 10 μm	W_2 = 20 μm
Z_{c2} = 29 ohm	Z_{c2} = 15 ohm
$Z_{\pi1}$ = 41 ohm	$Z_{\pi1}$ = 40 ohm
R_C = 41.78	R_C = 18.60
R_π = 0.98	R_π = 0.98
$\mathcal{V}_C$ = 128 M m/s	$\mathcal{V}_C$ = 124 M m/s
$\mathcal{V}_\pi$ = 102 M m/s	$\mathcal{V}_\pi$ = 102 M m/s

where W_1 and W_2 are the line widths of the middle and top conductors, respectively. The length of each pair of coupled lines is determined by using the average value of the phase velocities for c mode and π mode and is quarter wavelength at 14 GHz. We note that, for both coupled lines A and B, $R_\pi \approx 1$ and R_C is large. This justifies the use of approximation made in implementation of the synthesis procedure.

The ideal responses of this balun (without considering the difference in electrical lengths and the coupling) are shown in *Figure 8* and *Figure 9*. The actual responses of the synthesized circuit based on the exact model in *Figure 6* are also shown in these figures for comparison. The amplitude and phase at the two terminals of the balanced output port are also compared in *Figure 10 and Figure 11*.

From these results, we note that the simplification in the synthesis procedures (assuming $R_C \approx 1$, R_π is large and equal electrical lengths for all of the transmission line sections) yields a network with slightly lower bandwidth (about 10%) than the ideal design. Still, the suggested procedure provides a useful synthesis method.

IV. Concluding Remarks

A new model for general coupled transmission lines in inhomogeneous medium is derived without making any approximation. This model has been successfully used for synthesis of two-layer monolithic planar balun circuits for which no synthesis procedure has been available so far. The method is applicable for various other planar coupled line circuits and some of them will be reported at the Symposium.

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